Name: ______________________ Math 10F&PC H. **Date:** ____________

Chapter 7 System of Equations

Section 7.4: Using Substitution As a Strategy to Solve Linear System

PREREQUISITES: Solve for "x".

a) $x+6y=-2$

$-6y \quad -6y$

$x=-2-6y$ ✓

b) $3x+2y=8$

$\frac{3x}{3}=\frac{8}{3}-\frac{2y}{3}$

$x=\frac{8}{3}-\frac{2}{3}y$

PREREQUISITES: Solve for "y".

a) $-4x+y=11$

$y=11+4x$

b) $3x-2y=8$

$\frac{-2y}{-2}=\frac{8}{-2}-\frac{3x}{-2}$

$y=-4+\frac{3}{2}x$

In the previous sections, a linear system was solved by graphing out both equations and then finding the intersection point. This is often time consuming and also sometimes you can only find an approximate solution. We can use algebraic techniques to determine an exact solution. One method is called solving by SUBSITUTION. In this section, variables will eliminated by solving one equation for ONE variable (sometimes called ISOLATING the variable) , then SUBSTITUTING the result into the other equation.

Example 1: Solve the following system by substitution.

$4x-y=8 \longrightarrow \boxed{1}$

$x+2y=-7 \longrightarrow \boxed{2}$

To solve, you must decide on which variable to ISOLATE first! Let's do it two ways.

Solution #1: Choose to isolate "x".	*Solution #2:* Choose to isolate "y".
Eq. # 2 $x+2y=-7$ $x=-2y-7 \longrightarrow 2'$	Eq. # 1 isolate y $4x-y=8$ $-y=-4x+8$ $y=4x-8 \longrightarrow 1'$
Substitute the above expression into the other eqn. sub. 2' in 1 $4x-y=8 \longrightarrow 1$ $4(-2y-7)-y=8$ $-8y-28-y=8$ $-9y-28=8$ solve for (y) $\frac{-9y}{-9}=\frac{36}{-9}$ $\boxed{y=-4}$ Put it in Eq #2' sol. (1, -4) $x=-2(-4)-7=8-7=1$	*Substitute the above expression into the other eqn.* sub 1' in Eq. #2 $x+2(4x-8)=-7$ $x+8x-16=-7$ $9x-16=-7$ $\frac{9x}{9}=\frac{9}{9}$ $\boxed{x=1}$ $y=4x-8$ $y=4(1)-8=4-8=-4$ $\boxed{y=-4}$ Sol. : (1, -4) (1, -4)

It really doesn't matter which variable you isolate, the answers should be the same.

Even though it doesn't matter which variable you isolate, sometimes one is easier than another.

Example 2: Solve by substitution.

a) $5x+3y=5 \rightarrow (1)$

$2x+y=8 \rightarrow (2)$

- Isolate y From Eq. #(2)

$y = -2x+8 \rightarrow (2)'$

- Sub. 2′ in Eq. #(1)

$5x+3(-2x+8) = 5$

$5x-6x+24 = 5$

$-x+24 = 5$

$-x = -24+5$

$-x = -19$ $\therefore \boxed{x=19}$

Sub. $x=-19$ in 2′

$y = -2(19)+8 = -38+8$

$y = -30$ Solu: $(19, -30)$

b) $x+6y=9 \rightarrow (1)$

$3x-2y=-23 \rightarrow (2)$

isolate x From Eq. #(1)

$x = -6y+9 \rightarrow (1)'$

sub. (1)′ in (2)

$3(-6y+9)-2y=-23$

$-18y+27-2y=-23$

$-20y = -50$

$y = \frac{5}{2}$ or 2.5

$x = -6y+9$

$= -6\left(\frac{5}{2}\right)+9 = -15+9 = -6$

the solution is $\left(-6, \frac{5}{2}\right)$

c) $5y+2x=-2 \rightarrow (1)$

$5x-2y=24 \rightarrow (2)$

isolate x in Eq. #(1)

$5y+2x=-2$

$2x = -2-5y$

$x = \left(\frac{-5}{2}y-1\right) \rightarrow (1)'$

sub. (1)′ in Eq. #(2)

$5\left(\frac{-5}{2}y-1\right)-2y = 24$

$\frac{-25}{2}y - 5 - \frac{4}{2}y = 24$

$\frac{-29}{2}y - 5 = 24$

$\frac{-29}{2}y = 29$ x2

$\frac{-29y}{-29} = \frac{58}{-29}$ $\boxed{y=-2}$

SoL. $(4, -2)$ $x = \frac{-5}{2}(-2) - 1$

d) $8x+4y=1 \rightarrow \boxed{1}$

$7x=-2y \rightarrow \boxed{2}$

Isolate x in Eq. #$\boxed{2}$

$7x = -2y$

$x = \frac{-2}{7}y \rightarrow (2)'$

sub. (2)′ in (1)

$8\left(\frac{-2}{7}y\right)+4y = 1$

$\frac{-16}{7}y + 4y = 1$ x(7)

$-16y+28y = 7$

$12y = 7$ $\boxed{y = \frac{7}{12}}$

$x = \frac{-2}{7}\left(\frac{7}{12}\right) = \boxed{-\frac{1}{6}}$

SoL. $\left(-\frac{1}{6}, \frac{7}{12}\right)$

Chapter 7 System of Equations

Section 7.4: Using Substitution As a Strategy to Solve Linear System

e) $x - 8y = 24 \rightarrow ①$ $\boxed{x = 4}$
$2x - 16y = 30 \rightarrow ②$

Eq. #① isolate (x)

$x = 8y + 24 \rightarrow ①'$

Sub. ①' in Eq. #②

$2(8y + 24) - 16y = 30$

$\cancel{16y} + 48 - \cancel{16y} = 30$

$48 = 30$ Not true

No solution

$y = mx + b$

$-8y = -x + 24$

$y = \frac{1}{8}x - 3 \rightarrow \boxed{1}'$

$-16y = -2x + 30$

$y = \frac{1}{8}x - \frac{15}{8} \rightarrow \boxed{2}'$

Parallel lines never meet

f) $6x - 4y = 8 \rightarrow ①$
$9x - 6y = 12 \rightarrow ②$

① $\frac{6x}{6} = \frac{4y}{6} + \frac{8}{6}$

$x = \frac{2}{3}y + \frac{4}{3} \rightarrow ①'$

sub ①' in ②

$9\left(\frac{2}{3}y + \frac{4}{3}\right) - 6y = 12$

$6y + 12 - 6y = 12$

$12 = 12$ ✓

there are infinity of solution

$-4y = -6x + 8$

$y = \frac{3}{2}x - 2 \rightarrow \boxed{1}'$

$-6y = -9x + 12$

$y = \frac{3}{2}x - 2 \rightarrow \boxed{2}$

same line → infinity of solutions

Example 3: Solve this system by substitution. (What can we do about these fractions?)

$\frac{a}{2} + \frac{b}{3} = 1$ $\times 6 \rightarrow$ $3a + 2b = 6 \rightarrow ①$

$\frac{a}{4} + \frac{2b}{3} = -1$ $\times 12 \rightarrow$ $3a + 8b = -12 \rightarrow ②$

#① isolate (b)

$3a + 2b = 6$

$\frac{2b}{2} = \frac{-3a}{2} + \frac{6}{2}$

$b = \left(\frac{-3}{2}a + 3\right) \rightarrow ①'$

Sub. ①' in ②

$3a + 8\left(\frac{-3}{2}a + 3\right) = -12$

$3a - 12a + 24 = -12$

$-9a = -24 - 12$

$\frac{-9a}{-9} = \frac{-36}{-9}$

$\boxed{a = 4}$

sub $a = 4$ in ①'

$b = \frac{-3}{2}(4) + 3$

$b = -6 + 3 = -3$

Solution (a, b) $(4, -3)$

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Chapter 7 System of Equations

Section 7.4: Using Substitution As a Strategy to Solve Linear System

Example 4: Create equations to represent the following situations and solve using substitution. Remember to identify your variables first.

a) Janice invested \$2000, part in a bond that has a rate of return of 7% interest per year and the rest in a mutual fund that has a rate of return at 8% interest per year. After one year, the interest earned on the bond was \$50 more than the interest earned on the mutual fund. How much did she invest at each rate?

7%

8%

* let b - be the amount of money invested in bond.

let m - be ~ ~ ~ ~ ~ ~ mutual Fund.

intrest = (amount of money)(intrest rate)

7% b

8% m

Eq # ① $b + m = 2000 \rightarrow ①$

Eq # ② $7\% b = 8\% m + 50$

$0.07b = 0.08m + 50 \rightarrow ②$

Eq #① isolate b

$b = (2000 - m) \Rightarrow ①'$

sub ①' in ②

$0.07(2000 - m) = 0.08m + 50$

$140 - 0.07m = 0.08m + 50$

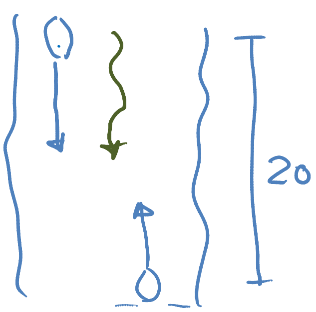
$140 - 50 = 0.08m + 0.07m \Rightarrow 90 = 0.15m$

$m = \frac{90}{0.15} = \boxed{\$600}$

$b = 2000 - m$

$= 2000 - 600 = \boxed{\$1400}$

b) Sue and Bill paddle 20 km upstream in 4 hours. The return trip takes 3 hours. What is the speed of the canoe relative to the water, and what is the speed of the current?



let x - be the speed of the canoe

let y - be the speed of the current

distance = speed × time.

* with the current

$3(x + y) = 20 \rightarrow \boxed{1}$

* upstream

$4(x - y) = 20 \rightarrow \boxed{2}$

$3x + 3y = 20 \rightarrow \boxed{1}$

$4x - 4y = 20 \rightarrow \boxed{2}$

Eq #② isolate x $\Rightarrow \boxed{2}$

$4x = 4y + 20$

$x = (y + 5) \Rightarrow \boxed{2}'$

sub $\boxed{2'}$ in $\boxed{1}$

$3(y+5) + 3y = 20$

$3y + 15 + 3y = 20$

$6y + 15 = 20$

$6y = 5$

$\boxed{y = \frac{5}{6} \text{ km/hr}}$

sub in ②'

$x = \frac{5}{6} + 5$

$\boxed{x = 5\frac{5}{6} \text{ km/hr}}$

Assignment: Page 427 Q# 4, 5, 8, 10, 12, 15, 19a &19b